SELF-SIMILAR SOLUTIONS OF THE SECOND KIND IN THE PROBLEM OF PROPAGATION OF INTENSE SHOCK WAVES

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G. I. BARENBLATT and G. I. SIVASHINSKII
(Moscow)

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The problem of propagation of intense shock waves is investigated in the case where the value of the adiabatic exponent γ_1 in the conditions at the shock wave differs from the value of this exponent γ in the differential equations describing the domain of continuous motion behind the wave. If $\gamma_1 < \gamma$ the proposed scheme makes possible approximate qualitative allowance for the energy expended on dissociation, ionization, and excitation of the vibrational degrees of freedom of the molecules. Solving the problem becomes a matter of constructing a self-similar solution of the second kind.

Self-similar solutions of the second kind are characterized by the fact that the exponents in the expressions of the self-similar variables in these solutions are obtained not on the basis of dimensional considerations, but rather as certain values which follow from the condition of existence of the solution in the whole [1]. As was shown in [2], solutions of the second kind arise as follows.

Self-similar solutions generally constitute asymptotic expressions of the solutions of more general, non-selfsimilar problems. They can be obtained from the non-selfsimilar solutions by taking certain limits as certain dimensionless combinations $\xi, \eta \dots$ containing the independent variables and the constant parameters of the problem go to zero or to infinity. If the corresponding limits exist and are finite, the limiting problem is associated with a self-similar solution of the first kind.

On the other hand, if a finite limit does not exist while the non-selfsimilar solution for $\xi, \eta \to 0$ has an asymptotics of one of the two "power" types

$$\eta^\alpha f(\xi), \qquad \eta^\alpha f(\xi / \eta^\beta) \tag{0.1}$$

where α, β are certain constants, then the limiting problem is associated with a self-similar solution of the second kind.

The exponents α, $\beta \dots$ remain in the limiting problem as a kind of trace of the non-selfsimilar solution from which the self-similar solution was obtained by taking its limit; this is what gives rise to the exponents of the self-similar variable which cannot be determined from dimensional considerations. This trace vanishes in the case of a regular limiting process, i. e. in the case of self-similar solutions of the first kind. It is essential that the irregularity of the limiting process be nonremovable (i. e. that its elimination cannot be effected by, let us say, the application of conservation laws); if this is not so, then the self-similar solution can be obtained as a solution of the first kind by a transformation of the dimensionless variables.

In [2] we analyzed an example of a problem of unsteady filtration in which a self-similar solution of the first kind resulted for one value of a parameter occurring in the problem, while a self-similar solution of the second kind resulted for all the other values. The asymptotics of the non-selfsimilar solution took the form of the first expression of (0.1). A similar situation arises in the problem investigated in the present paper, although the asymptotics of the non-selfsimilar solution in this case takes the form of the second expression of (0.1). Our problem concerns the propagation of intense shock waves, although in contrast to the familiar formulation of the intense-explosion problem first proposed by Sedov in [3] (see also [1, 4, 5, 6, 7]), we assume that the value of the adiabatic exponent γ_1 in the conditions at the shock wave differs from the value γ of this exponent in the differential equations describing the domain of continuous motion behind the wave.

For $\gamma_1 < \nu$ this scheme makes possible approximate qualitative allowance for energy expenditure on dissociation, ionization, and excitation of the vibrational degrees of freedom of the molecules in a certain intermediate range of propagation of the intense shock wave (cf. [8]). It appears that in dealing with the problem of propagation of intense blasts in an atmosphere containing fine dust particles and water droplets (e.g. in a cloud) it is also advisable to allow for the difference between the adiabatic exponents at the shock and in the continuous portion of the flow, since the dust particles or water droplets are carried away by the shock wave or are burned up or vaporized at the latter.

In this connection we must mention studies [9, 10, 11]. Sidorkina [11] (see also [5]) investigated the propagation of intense blasts in aerosols, reducing the problem to the investigation of intense shock waves in a polytropic state when the polytropic exponents at the shock wave and in the continuous portion of the flow must be taken equal to each other but not equal to the Poisson adiabatic exponent of the gas. The present study differs fundamentally from [9–11] in the fact that the presence of the energy integral in the problems considered in the latter papers yields self-similar solutions of the first kind.

For completeness we consider the cases $\gamma_1 < \gamma$ and $\gamma_1 > \gamma$. The resulting family of solutions describes a continuous set of motions which includes ordinary intense blasts and the propagation of intense detonation waves.

1. Let us begin by considering the following non-selfsimilar problem. An infinite space filled with quiescent gas of density ρ_0 contains a spherically symmetric domain of diameter d in which a finite energy E is released instantaneously at the initial instant $t = 0$. As usual in problems involving intense shock waves, the initial gas pressure is negligible.

As stated above, we assume that the adiabatic exponent in the domain of continuous motion is equal to γ, and that the adiabatic exponent at the shock wave front is $\gamma_1 \neq \gamma$, so that the conditions at the shock wave can be written as [5]

$$\rho_2 (u_2 - c) = -\rho_0 c, \qquad \rho_2 (u_2 - c) u_2 + p_2 = 0$$

$$\rho_2 (u_2 - c)\left(\frac{u_2^2}{2} + \frac{1}{\gamma_1 - 1}\frac{p_2}{\rho_2}\right) + p_2 u_2 = 0 \tag{1.1}$$

Hence,

$$u_2 = \frac{2}{\gamma_1 + 1} c, \quad \rho_2 = \frac{\gamma_1 + 1}{\gamma_1 - 1}\rho_0, \quad p_2 = \frac{2}{\gamma_1 + 1}\rho_0 c^2 \tag{1.2}$$

Here c is the velocity of propagation of the shock wave, ρ is the gas density, p is the pressure, and u is the velocity; the subscript 2 denotes the values of the variables directly behind the wave front.

The integral

$$\int_0^\infty \rho\left(\frac{u^2}{2} + \frac{p}{(\gamma - 1)\rho}\right) r^2 dr$$

is not preserved in this problem if $\gamma_1 \neq \gamma$. This is because the third condition of (1.1), which can be rewritten as

$$\rho_2 (u_2 - c)\left(\frac{u_2^2}{2} + \frac{1}{\gamma - 1}\frac{p_2}{\rho_2}\right) + \\ + p_2 u_2 - \frac{\gamma_1 - \gamma}{(\gamma - 1)(\gamma_1 - 1)}\left(\frac{p_2}{\rho_2}\right)\rho_2 (u_2 - c) = 0 \qquad (1.3)$$

corresponds formally to the loss (for $\gamma_1 < \gamma$) or influx (for $\gamma_1 > \gamma$) of energy at the shock wave front. There is, of course, no actual loss of influx of energy, but merely the conversion of energy into other forms. Thus, the defining parameters of the problem are ρ_0, E, r, t, d, γ, γ_1 (where t is the time and r is the distance from the center of symmetry), so that the problem contains two independent parameters, namely

$$\xi = r\left(\frac{Et^2}{\rho_0}\right)^{-1/5}, \qquad \eta = d\left(\frac{Et^2}{\rho_0}\right)^{-1/5} \qquad (1.4)$$

Making use of Sedov's variables [5], we can express the velocity, density, and pressure as

$$\rho = \rho_0 R(\xi, \eta, \gamma, \gamma_1), \quad u = \frac{r}{t} V(\xi, \eta, \gamma, \gamma_1), \quad p = \rho_0 \frac{r^2}{t^2} P(\xi, \eta, \gamma, \gamma_1) \quad (1.5)$$

The familiar solution of the problem of an intense point explosion for $\gamma = \gamma_1$ is the solution of the limiting problem with singular initial data corresponding to $d = 0$ and obtainable as the limit of solution (1.5) as $\eta \to 0$,

$$R = R(\xi, \gamma), \quad V = V(\xi, \gamma), \quad P = P(\xi, \gamma) \\ r_0(t) = \xi_0(\gamma)(Et^2 / \rho_0)^{1/5} \qquad (1.6)$$

Here $r_0(t)$ is the radius of the shock wave and ξ_0 is a constant which depends on γ.

Superficially, it would appear that a similar limiting form of solution (1.6) must also hold for $\gamma_1 \neq \gamma$, since all the considerations of dimensional analysis for the non-selfsimilar solution remain valid (the appearance of a new parameter γ_1 does not alter the situation in this respect) and since taking the limit as $d \to 0$, i.e. as $\eta \to 0$ (passage to a point explosion), must render the solution independent of the parameter η, and, seemingly, make it a self-similar solution of the form (1.6). However, a self-similar solution of the form (1.6) does not exist for $\gamma_1 \neq \gamma$. This can be inferred in elementary fashion from Sedov's proof of the existence of a solution of the intense-explosion problem for $\gamma_1 = \gamma$ [5].

In fact, the following relation is valid for a solution of the form (1.6) which satisfies the symmetry relation $u(0, t) = 0$ for all ξ $(0 \leqslant \xi \leqslant \xi_0)$:

$$P = \frac{\gamma - 1}{2\gamma}\frac{RV^2(V - 2/5)}{(2/5\,\gamma - V)} \qquad (1.7)$$

Making substitutions (1.6) in conditions (1.2) at the shock wave, we obtain

$$\frac{2P}{\gamma_1 - 1} = RV^2, \quad P = \left(\frac{2}{5} - V\right)RV, \quad P = \frac{2}{5}V \quad \text{for } \xi = \xi_0 \qquad (1.8)$$

These relations are not compatible for $\gamma_1 \neq \gamma$, which proves the nonexistence of a solution of the form (1.6) for $\gamma_1 \neq \gamma$.

It is natural to expect, however, that for a sufficiently small diameter d of the domain in which the energy is released and a sufficiently high energy, the dependence of the motion on this diameter must soon cease (i.e. the motion becomes self-similar in the limiting case). This would seem to lead inevitably to a solution of the form (1.6). In fact this is not the case, and the limiting solution, which is indeed self-similar, cannot be expressed in the form (1.6).

The above analysis of the case $\gamma_1 \neq \gamma$ proved insufficient (since a self-similar solution in the expected form does not exist) because our assumption of the existence of finite limits of the functions R, V, P as $\eta \to 0$ was not valid.

2. In constructing the limiting solution we shall be concerned essentially with the asymptotic form of the solution of the above non-selfsimilar problem for large t. In principle, this solution can be either self-similar or non-selfsimilar. Both ξ and η tend to zero with increasing t. In accordance with the general considerations developed in [2], we assume that even though a finite limit does not exist as ξ and η tend separately to zero, there does exist a number β such that the dominant terms of the asymptotic forms of the functions P, V, R as $\xi, \eta \to 0$ are given by

$$P = P(\xi/\eta^\beta), \qquad R = R(\xi/\eta^\beta), \qquad V = V(\xi/\eta^\beta) \tag{2.1}$$

(Case 2 of [2]) (*).

If the above assumption is valid, then the limiting motion belongs to the class of power-law self-similar solutions of the gas dynamic equations pointed out in [12, 13, 5, 14]. It is convenient to renormalize the independent self-similar variable as in [5] and to take it in the form

$$\zeta = \mathrm{const}\,\frac{\xi}{\eta^\beta} = r\left(\frac{At^2}{\rho_0}\right)^{-(1-\beta)/5}, \qquad A = \sigma E d^{5\beta/(1-\beta)} \tag{2.2}$$

where the constant parameter σ is chosen in such a way that $\zeta = 1$ at the shock wave front. This means, among other things, that the asymptotic law of shock wave propagation can be written as

$$r_0 = A^{(1-\beta)/5}\rho_0^{-(1-\beta)/5}t^{(2-2\beta)/5} \tag{2.3}$$

The parameters ξ and η can be made to go to zero for fixed r and t by making E and d go to infinity and to zero, respectively. Moreover, if our assumption is valid, then the self-similar limiting motion (the "point explosion") corresponds for $\gamma_1 \neq \gamma$ to the release at the blast center not of a finite amount of energy, but rather of an energy which tends to infinity or to zero as $d \to 0$ in such a way that

$$Ed^{5\beta/(1-\beta)} = \mathrm{const} \tag{2.4}$$

Thus, the limiting motion is a self-similar motion of the second kind [1]. The exponent β, or (which is the same thing) the exponent of the time in the expression for the self-similar variable, namely $\alpha = {}^2/_5 - {}^2/_5\,\beta$ remains as the "trace" of the limiting passage from the initial non-selfsimilar motion. This trace vanishes in the ordinary self-similar solutions (and also in our problem when $\gamma_1 = \gamma$). The exponent in question must be determined from the condition of existence of a self-similar solution of the problem in the whole.

Let us obtain the solution of the problem by the method of Chernyi [15, 1] which will serve us as a rough preliminary estimate. In the zeroth approximation the entire mass $M = (4\pi/3)\rho_0 R^3$, of gas taking part in the motion is concentrated at the shock wave front with the radius $R(t)$; the gas pressure p_c in the cavity is given by $p_c = \varepsilon p_2$, where p_2 is the pressure at the front and ε is a constant which must be determined. As in the ordinary case $\gamma_1 = \gamma$, the equation of momentum implies that

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*) In principle this case admits of expression of the form $P = \xi^\delta P(\xi/\eta^\beta)$ (cf. (0.1)), etc., but the equations of motion imply that δ and the similar constants in the expressions for the other variables are equal to zero.

$$c = aR^{-3(1-\varepsilon)}, \qquad a = \text{const} \tag{2.5}$$

The enregy E of the gas taking part in the motion is given by (2.6)

$$E = \frac{4\pi}{3} R^3 p_c \frac{1}{\gamma - 1} + M \frac{\dot{u_2}^2}{2} = \frac{4}{3}\pi R^3 \rho_0 c^2 A, \qquad A = \frac{2}{\gamma_1 + 1}\left[\frac{\varepsilon}{\gamma - 1} + \frac{1}{\gamma_1 + 1}\right]$$

By virtue of relations (1.3), (1.1) and (1.2) we have

$$\frac{dE}{dt} = -4\pi R^2 \rho_0 c \tau \frac{p_2}{\rho_2} = -\tau 4\pi R^2 \rho_0 c^3 \frac{2(\gamma_1 - 1)}{(\gamma_1 + 1)^2}, \qquad \tau = \frac{\gamma - \gamma_1}{(\gamma - 1)(\gamma_1 - 1)} \tag{2.7}$$

Substituting (2.5) and (2.6) into (2.7), we obtain the following equation for ε :

$$\varepsilon^2 + \varepsilon\left[\frac{\gamma - 1}{\gamma_1 + 1} - \frac{1}{2}\right] - \frac{(\gamma + \gamma_1 - 2)}{2(\gamma_1 + 1)} = 0 \tag{2.8}$$

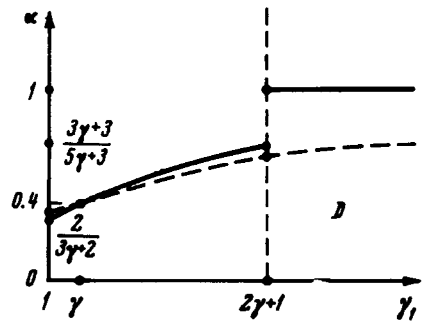

Fig. 1

In the self-similar mode $R \sim t^\alpha$, $c \sim t^{\alpha-1}$. Hence, $\alpha = 1/(4 - 3\varepsilon)$. We see that the function $\alpha(\gamma_1)$ can be determined by the Chernyi method uniquely for all $\gamma_1 > 1$; it appears as the broken curve in Fig. 1, where D is the domain of nonuniqueness of the exact solution.

3. Let us consider the exact solution. To determine the parameter α and to construct the limiting solution for $\gamma_1 \neq \gamma$ we substitute (1.5)–(2.1) into the gas dynamics equations and conditions (1.3) at the shock wave. In accordance with [5] this yields

$$\frac{dz}{dV} = \frac{z}{\Delta}\{[2(V-1) + 3(\gamma - 1)V](V - \alpha)^2 - (\gamma - 1)V(V-1)(V - \alpha) -$$
$$- [2(V-1) + \varkappa(\gamma - 1)]z\} \tag{3.1}$$

$$\Delta = (V - \alpha)[V(V-1)(V-\alpha) + (\varkappa - 3V)z], \qquad \varkappa = \frac{2(1-\alpha)}{\gamma}, \qquad z = \frac{\gamma P}{R}$$

$$\frac{d\ln\zeta}{dV} = \frac{z - (V-\alpha)^2}{V(V-1)(V-\alpha) + (\varkappa - 3V)z} \tag{3.2}$$

$$(V - \alpha)\frac{d\ln R}{d\ln\zeta} = -3V - \frac{V(V-1)(V-\alpha) + (\varkappa - 3V)z}{z - (V-\alpha)^2} \tag{3.3}$$

The conditions of conservation of the material flux, momentum, and energy at the shock wave take the form

$$z = -\gamma V(V - \alpha), \quad Rz = \alpha\gamma V \qquad \frac{2z}{\gamma(\gamma_1 - 1)} = V^2 \quad \text{for } \zeta = 1 \tag{3.4}$$

The exponent $\alpha = {}^2/_5 - {}^2/_5\beta$ can be determined from the condition whereby the integral curve in the plane zV of Eq. (3.1) which emerges from the point M defined by relations (3.4) (the image of the shock wave front) passes through the point N (the image of the center of symmetry); in moving from N to M the self-similar independent variable ζ increases monotonically from $\zeta = 0$ to $\zeta = 1$.

Analysis shows that the dependence of α on γ_1 is of the form indicated by the solid curve in Fig. 1. In the interval from $\gamma_1 = 1$ to $\gamma_1 = 2\gamma + 1$ it is represented by the monotonically increasing curve $\alpha(\gamma_1)$ which passes through the points

$$(\alpha = {}^2/_3\gamma + 2, \gamma_1 = 1), \qquad (\alpha = {}^2/_5, \gamma_1 = \gamma)$$
$$(\alpha = (3\gamma + 3) / (5\gamma + 3), \quad \gamma_1 = 2\gamma + 1)$$

The exponent α is not uniquely defined for $\gamma_1 > 2\gamma + 1$. A similar ambiguity was noted in [16] for another self-similar problem of the second kind (that of a convergent shock wave). The author of the latter paper suggested that in reality α assumes its minimum value. Whether this hypothesis is valid remains unclear, however.

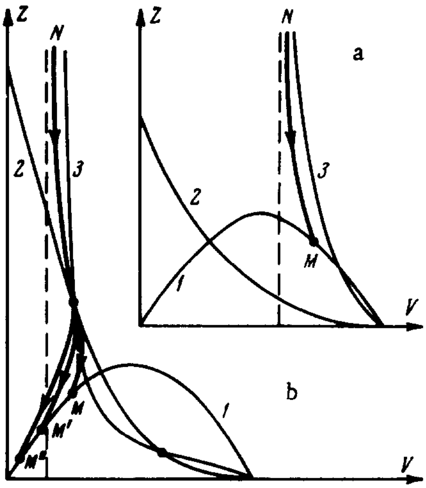

Fig. 2

The qualitative form of the required integral curve in the plane zV in the case $\gamma_1 < < \gamma$ (for $\gamma < 2$) is shown in Fig. 2a, where the numbers 1–3 refer to the curves

$$z = -\gamma V (V - \alpha), \; z = (V - \alpha)^2,$$
$$z = V (V - 1) (V - \alpha) (3V - \varkappa)^{-1}$$

respectively.

If $\gamma_1 > \gamma$ and $\gamma_1 > 2\gamma + 1$ the integral curve which emerges from the saddle and passes through a singular point of the node type, namely the left-hand point of intersection of the curves

$$z = V (V - 1) (V - \alpha) (3V - \varkappa)^{-1},$$
$$z = (V - \alpha)^2$$

yields not just one, but a whole family of integral curves satisfying all the above conditions. Thus, as soon as a node-type point arises in the plane zV there appears a certain interval of values of α, and the value of the exponent α becomes nonunique (Fig. 2b).

The analysis in the intermediate case $\gamma < \gamma_1 < 2\gamma + 1$ is quite similar to that carried out in the first case, and α can be determined unambiguously for each γ_1; here we have

$$\alpha(2\gamma + 1 - 0) = (3\gamma + 3) (5\gamma + 3)$$

4. 1°. As in the case of self-similar solutions of the second kind in general, the solution in question is defined to within a constant σ. In the case $\gamma_1 = \gamma$, $A = \sigma E$ the constant σ can be determined [5] from the conservation law

$$\int_0^\infty \rho \left(\frac{u^2}{2} + \frac{p}{(\gamma - 1)\rho} \right) r^2 dr = \text{const} \tag{4.1}$$

which also holds for non-selfsimilar motions. No such conservation law applies when $\gamma_1 \neq \gamma$, and the only way of determining σ is by numerical calculation of the non-selfsimilar problem until the emergence of the solution onto the above self-similar asymptotics, which is known to within the required constant.

2°. Analysis of the range $\gamma_1 > 2\gamma + 1$, and in particular the elimination of the ambiguity which prevails in this case is beyond the scope of the present paper. We merely note the following facts. For $\alpha < 1$ the velocities are everywhere directed outward from the center of symmetry. For $\alpha > 1$ the flow zone is divided into two parts by a certain sphere: inside this sphere the velocities are directed towards its center; outside it they are directed outward from the center. The case $\alpha = 1$ corresponds to intense detonation waves (*). In the case of an intense detonation the condition at the shock wave is of the form

$$\frac{1}{\gamma(\gamma-1)}z - \frac{1}{2}V^2 = q \tag{4.2}$$

where q is the ratio of the heat generated at the front per unit mass of the gas to the square of the velocity of propagation of the wave relative to the stationary gas. Here we have already made allowance for the fact that the independent self-similar variable is inversely proportional to the time and that the detonation wave propagates at a constant velocity. As we know [18], in order to obtain a unique solution of the detonation problem it is necessary to impose an additional condition (usually the Chapman-Jouguet condition) consisting of the requirement that the velocity of propagation of the wave relative to the gas be equal to the local velocity of sound, so that for $\zeta = 1$ we have

$$z = (V-1)^2 \tag{4.3}$$

The last condition of (3.4) can be rewritten as

$$\frac{z}{\gamma(\gamma-1)} - \frac{1}{2}V^2 = z\frac{\gamma_1-\gamma}{\gamma(\gamma-1)(\gamma_1-1)} \tag{4.4}$$

From (4.3) and (4.4) we infer that for every solution of the detonation problem there exists a γ_1 for which this solution coincides with the solution of the problem under investigation for $\alpha = 1$. However, this γ_1 is bounded below. Let us now determine the γ_1, corresponding to the mode in which the Chapman-Jouguet condition holds. Solving system (4.3), (4.4) simultaneously with the first condition of (3.4) for $\alpha = 1$, we find that

$$z = z^* = \gamma^2(\gamma+1)^{-2}, \qquad q = q^* = {}^1/_2\,(\gamma^2-1)^{-1}$$

at the detonation front, so that

$$\gamma_1 = 2\gamma + 1$$

The remaining "partly compressed" modes clearly correspond to $q < q_*$: the Chapman-Jouguet mode is associated with the minimum wave propagation velocity for equal rates of heat release at the wave front. These modes correspond to $\gamma_1 > 2\gamma + 1$; the velocity of the products of combustion behind the detonation wave front for these solutions is larger than the velocity of sound.

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*) A point explosion of a fuel gas mixture is considered in (17), where the reader will find further references to the literature.

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ON THE DIFFRACTION OF SHOCK WAVES

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G. M. ARUTIUNIAN
(Moscow)

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It is shown that for sufficiently large values of the adiabatic exponent of a gas the solution of the problem of diffraction of an arbitrarily intense shock wave at a small angle is of a form different from the usual one, and that the solution of this problem takes three forms in the general case. The corresponding pressure formulas for the case in question are derived.

The problem of diffraction of an arbitrarily intense shock wave at a small angle was first investigated by Lighthill [1], who, however, did not go so far as to obtain complete analytic solution. This was one of the factors which led Ting and Ludloff [2] to reconsider the problem using a different method of solution. These authors succeeded in